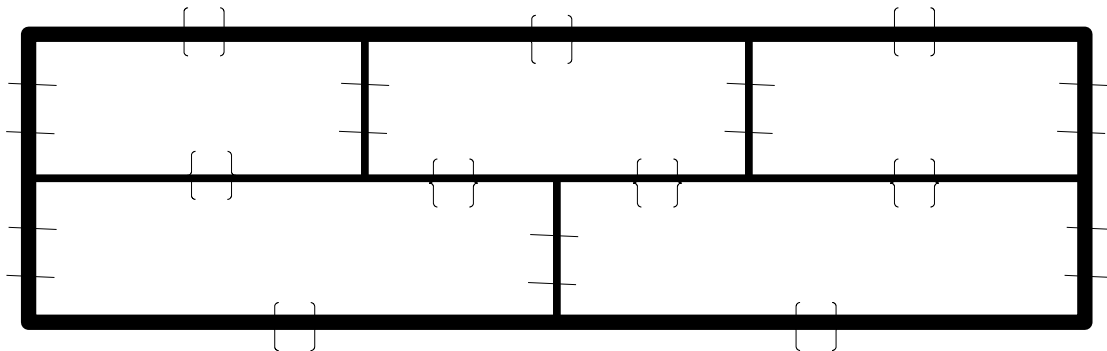


The House Problem (A Study in Solving Trace Problems)

1. Definition and Identification of Main Crux of the Problem

The following “House Problem” cannot be solved with the number of rooms and doors given. The main problem then becomes how to identify the reasons for it being unsolvable since a lot of trail and error is an inefficient method and wastes time.

Fig 1.1



This is a Floor Plan with Doors. You must pass through each door only once, and you must pass through all doors without retracing back on yourself or 'bouncing off' any doors. You can start outside of the house or inside, in any room you wish.

At first glance, it would seem the only way to solve this would be trial and error by getting a pen or pencil and starting to draw a line right away. Unfortunately, this could take you a very long time to come to the eventual conclusion that it is impossible to solve. **The question, then, is how can we find this out without all the trial and error work which may still leave the question unanswered?**

2. Problem Analysis

It seems reasonable that the number of doors and rooms are crucial. If we start in a room with only 2 doors, it's logical we must return to that room again in order to complete the problem and finish going through all the doors. If it has 3 doors we would need to leave, come back, and leave again in order to finish the problem and go through all of the doors. Let's keep in mind that we'd like to determine the significance of this.

Now let's try an intersection with 3 lines coming from them. (See Figure 2.4 to the right.)

Note that after drawing line 2 we are Offered two choices, either straight up or diagonally up and left.

It makes no difference which way is taken since the path allows you to complete the problem either way.

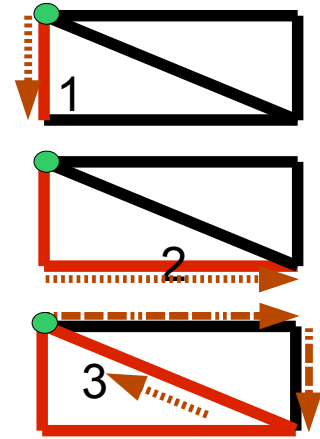
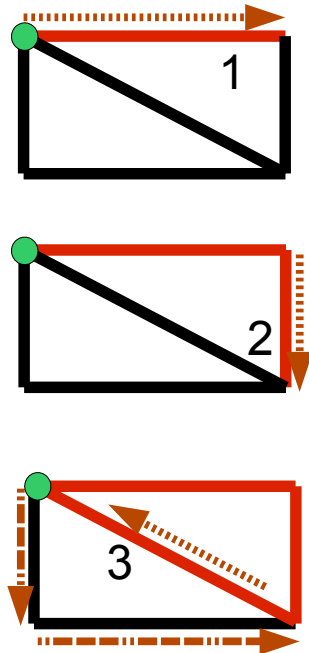


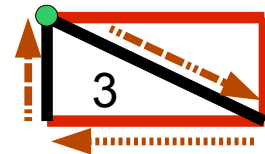
Fig. 2.4

Just to make sure, let's try it again, going in a different direction from the starting point, and instead of going down we will travel to the right. Again, from the same 3-line intersection (Fig 2.5)

Fig. 2.5



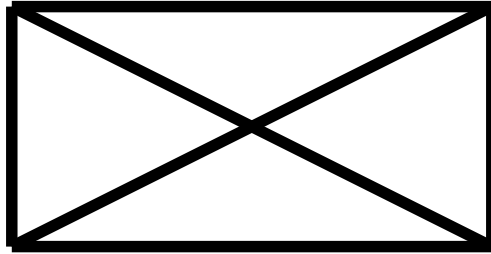
Again, it makes no difference which way is taken when faced with a choice of paths to take. Either path taken after drawing line 2 will allow you to complete the puzzle problem.



The image above shows the outcome of a different path.

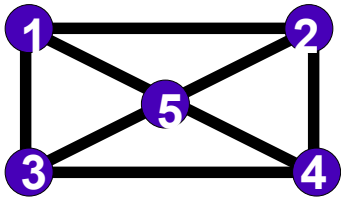
This is a very important discovery! We can finish this puzzle, but only if we choose to start at an intersection with 3 lines coming from it. If we start at an intersection with only 2 lines coming from it, we will most likely fail. **Therefore, a problem with 2 odd numbered intersections and 2 even numbered intersections is solvable.** But let's not stop there. We've come up with some useful information, but it still isn't enough to help us solve the original unsolvable problem. So let's take the image from figure 2.2 and just draw an additional line through it....

Fig. 2.6



This trace problem in Figure 2.6 is an unsolvable problem. Why does the extra line make it impossible to be solved? That is what we need to find out. This is like the original problem in that it cannot be solved. To help understand this, analyzing the attempt as it progresses will be illustrated. I will start by counting the intersections and lines. (See Figure 2.7)

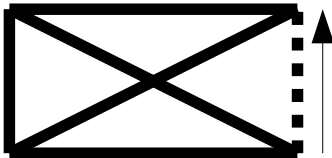
Fig. 2.7



Intersection 1 = 3 Lines = Odd
 Intersection 2 = 3 Lines = Odd
 Intersection 3 = 3 Lines = Odd
 Intersection 4 = 3 Lines = Odd
 Intersection 5 = 4 Lines = Even

There are 4 Odd Intersections and 1 Even Intersection. We will begin tracing and start from an odd numbered intersection. Because of the geometry of this problem, it does not matter which of the 4 odd numbered intersection is chosen. So we'll choose the bottom-right intersection. (See Figure 2.x)

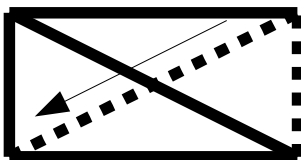
Fig. 2.8



Now I am going to analyze this move to see what the results of moving from one odd intersection to another are. Since the starting point left an odd intersection, it is a fact that at some point we will have to return to it again AND leave it again in order to complete tracing all of the lines.

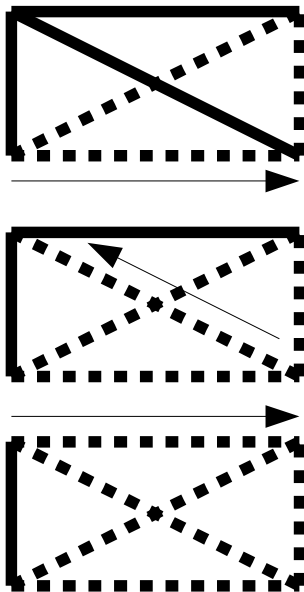
It is also a fact that the intersection that was just arrived at needs to be left AND returned to in order to complete tracing all of the lines. (See figure 2.x below) So far it looks possible, so continuing on may reveal more information.

Fig. 2.9



This move does not reveal anything interesting, but it does show us two routes, or choices, that can be taken. If the line is traced straight up and right, we run into a dead end. If we go straight up and go diagonally down to the right bottom, again a dead end is evident. So that leaves one logical way to travel.

Fig. 2.10



Here is something notable; I have again arrived where I started. But then this also reveals the futility of going further. If the trace is continued, there is only two directions which go opposite directions, each leaving the problem unsolved.

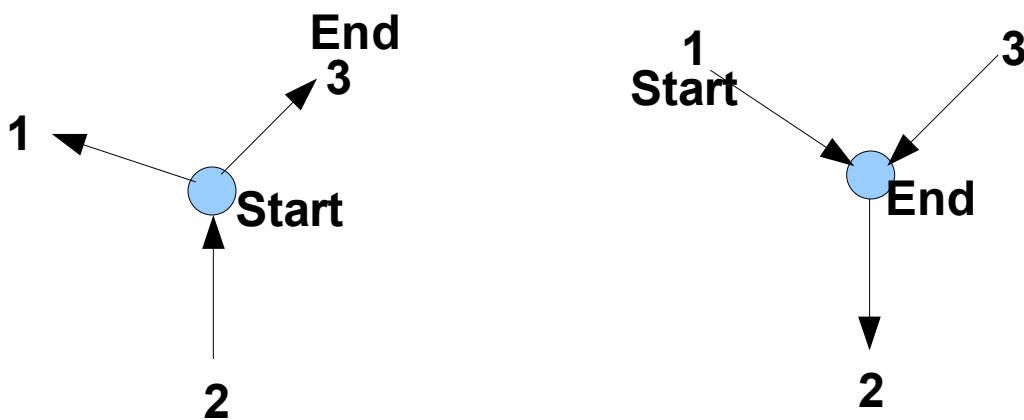
No matter what line is traced, the outcome will always be that there will be one line left which cannot be traced. Even using the even numbered intersection makes no difference.

Using logical thought with what we know about odd numbered intersections, the reason why it cannot be done might be more apparent. So a review of the odd numbered intersection will be helpful. We know if we start at an intersection that has three lines, we need to leave, come back, and leave again.

We also know that if we do not start at an intersection that has three (3) lines, we need to come into the intersection, leave it, and return to it again in order to complete the trace problem. BUT if return back to it again, it is seen that the line leading to it must also be the very last line traced at the end of solving the trace problem.

Therefore if we do not start at the odd intersection, whether it be 3, 5 or 7 odd number of lines, eventually that must be the last intersection to be completed in this particular problem. And contrarily, if we do start at the odd numbered intersection, we are able to continue on through that intersection as the last line of that intersection is traced. The diagrams in Figure 2.11 below demonstrates this.

Fig. 2.11



We can now add the fact that 4 Odd intersections is unsolvable to the fact that 2 Odd intersections is solvable. What about the even? It does not matter, since the result is always either coming back to or continuing through such an intersection. Consider the illustration in Figure 2.8 below. You can see that if you go into an even numbered intersection, the only result is having to go back out of it again. Alternatively, if you go out from that intersection, you will have to return to it again. In other word, you return to or continue on through this intersection no matter how many even intersections (as shown below with 2 and 3 even intersections.).

Note that even with 4 lines coming from the intersection, the end result will be the same. Therefore the even numbered intersections make no significant difference to solving the problem other than if you start there, you must in return end there. That's if you use an even intersection as a starting point.

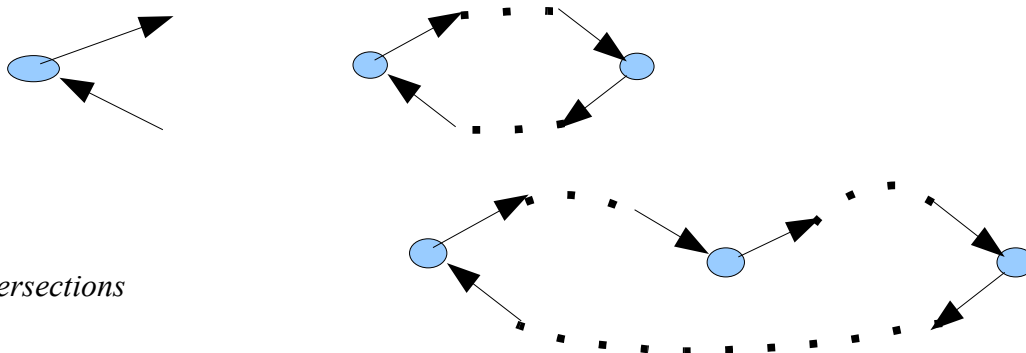


Fig. 2.12

Even Numbered Intersections

Reviewing what we have so far, we know the following facts...

1. It does not matter how many even intersections there are
2. A problem with 2 Odd intersections is solvable.
3. A problem with 4 Odd intersection is NOT solvable.

So let's break down the original problem to see what we have. Remember instead of intersections, the original problem has rooms, and instead of lines to trace, it is doors to go in and out of. Count the rooms and doors to see how many even and odd intersections we have. See Figure 2.13 below for the visual breakdown.

F 9

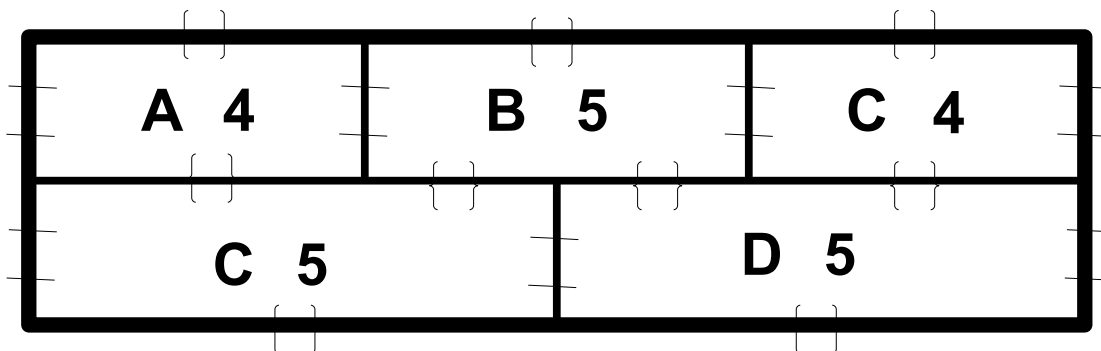


Fig. 2.13

This makes 3 Odd rooms. We already know for a fact that this is an unsolvable problem. And since it is unsolvable and has 3 Odd rooms (Or intersections), we can complete our initial analysis with the following facts now...

1. It does not matter how many even intersections there are
2. A problem with 2 Odd intersections is solvable.
3. A problem with 3 Odd intersections is NOT solvable.
4. A problem with 4 Odd intersection is NOT solvable.

So there we have a basic, albeit not fully tested, generalized set of rules to use when trying to solve either House Problems or Line-Trace Problems without having to go through all of the guesswork involved by trial and error. Simply by using these 4 basic guidelines, we can now make a much easier and faster deduction as to whether the problem is or is not solvable.

3. Heuristics (Methods) Used in Problem Analysis

Some of the methods we used in the analysis are characterized as follows. These methods are called Heuristics, and are valid ways for finding solutions to otherwise seemingly complex problems. There are more, however I am only listing the ones used here.

- a) Drawing a Diagram
- b) Eliminating Possibilities
- c) Looking for a Pattern
- d) Identify Sub-Problems
- e) Analyze the Units
- f) Solve/Use an Easier Related Problem
- g) Evaluate Finite Differences

With these heuristics we were able to come up a pretty good solution to the main problem, which was not how it could be solved, but why it can not be solved. However, keep in mind that this may not be the only solution since we did not analyze beyond 4 Odd numbered intersections (Called Vertices (Plural) or Vortex (singular).) But this is definitely good enough to solve the problem of taking too long to solve a problem with 4 or less odd vertices.

4. Summation, Review of the Problem and Analysis

Looking back at this entire process, we discovered four facts useful for assembling a set of guidelines to use in answering the main problem. That main problem was how to know if it can even be solved without having to take all of the time and guesswork of trial and error efforts. As a benefit, if you were to need to plan a route through your neighborhood delivering something (mail, newspapers, ice cream) then you can plan a very fuel efficient route to take.

In order to be able to decide, there are now 4 basic rules to help guide you in determining the answer when viewed in this manner. First, we started analyzing some basic facts. Then we simplified the problem and tested some theories with odd numbered intersection (or vertices.). Finally we concluded we could determine the answer from the final fact that the original problem was unsolvable, and counted how many odd intersections that problem had.

Here is a review of the four facts we discovered along the way...

1. It did not matter how many even intersections there were.
It does not matter if it is 2,4,6 or 8. But if you start there, you must also end there.
2. A problem with 2 Odd intersections was solvable!
We tested that and found this condition to be true.
3. A problem with 3 Odd intersections was NOT solvable.
When we added a line to our example, it was found to be unsolvable. (Rectangle with X inside of it.)
4. A problem with 4 Odd intersection was NOT solvable.
This fact was taken from the original problem,

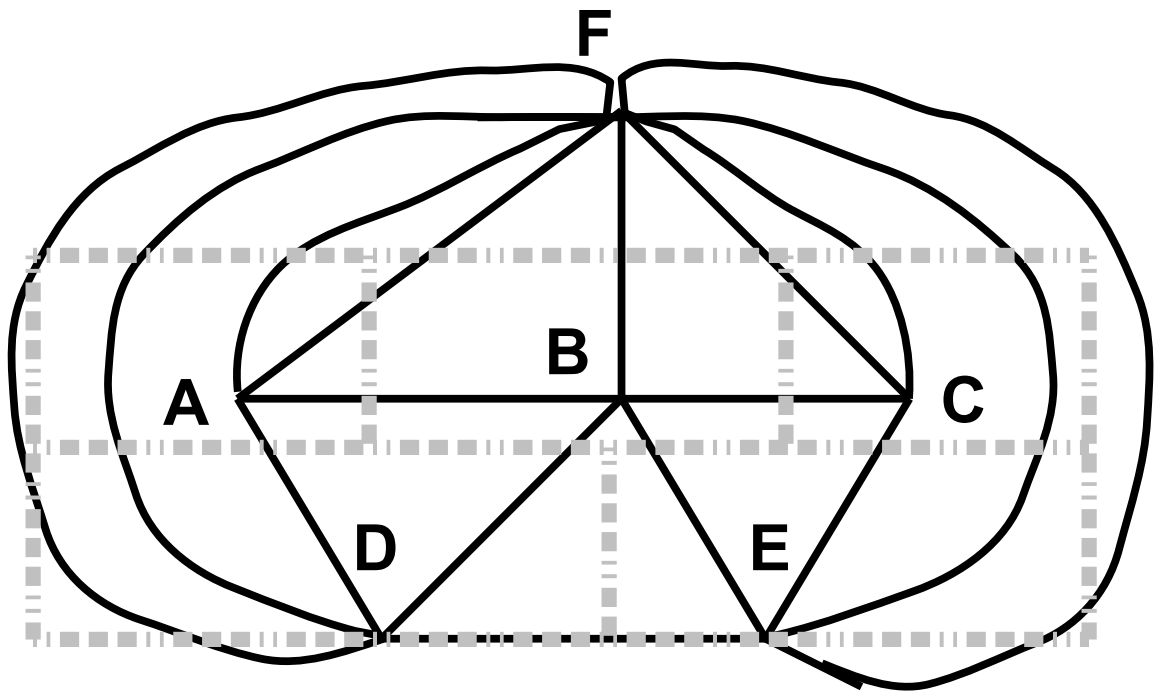
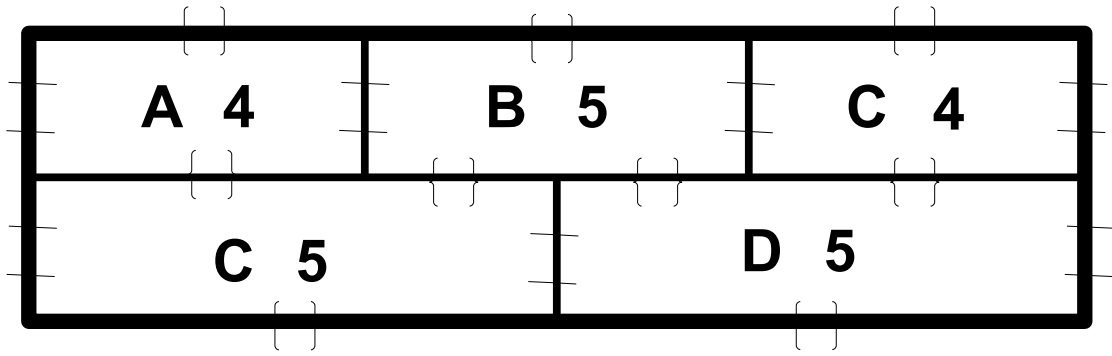
Using these simplified guidelines to specify the answer are acceptable for most less advanced problems of the same type (House or Trace) and can use further study for more advanced problems of this kind. It relates to networking as much as it does to city streets, and poses logistical situations to be overcome using mathematical and logical means.

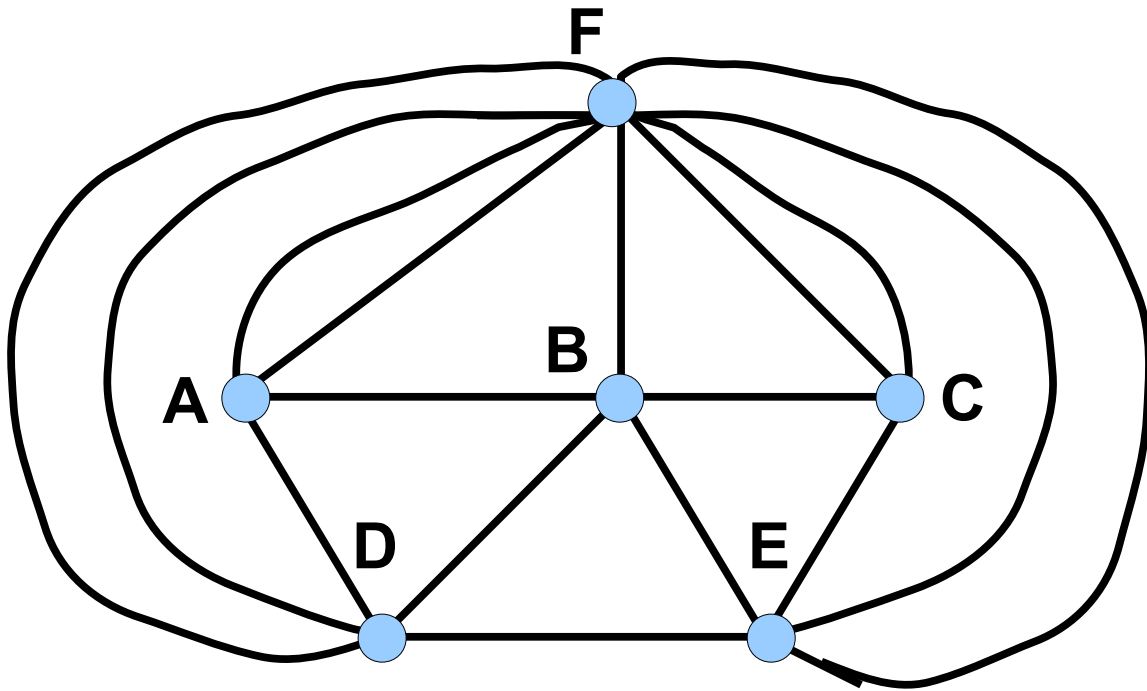
Another angle is evaluating the room versus intersection (house vs trace) problems with the rooms explaining the outside area and it's conversion to the trace-style problem. A comparison and evaluation of these may produce interesting results with which to modify our list of facts for determining a solvable/unsolvable problem.

For more completeness, Figure 4.1 demonstrates the method used for converting from a house style problem to a trace style problem using the original unsolvable problem from Figure 1.1 as an example...

Fig 4.1

F 9





Although not a pretty result, this IS the Trace Problem you would have to go through in order to come up with a solution (or not in this case.) However, we DO know enough now to tell us rather quickly, using the guidelines we had set up throughout solving this original problem of why this would be an unsolvable problem, that this is a problem that is one which is not solvable.

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